GeoJournal 21.3 223–231

A Theoretical Model for Average River Runoff

Iberall, A. S., Scholar, UCLA, Oral Biology, Los Angeles, CA 90024-1668, USA

Based on a physical theory for the hydrological cycle, for the time averaged steady state in which the precipitation equals evaporation (commonly called evapotranspiration) plus runoff, and the assumption that an intrinsic coordinate map of a large land mass can be set up as a projection on the Earth's geoid of altitude contours vs. a trace of the position of rivers, such a map can be used to predict or estimate the average runoff in rivers if the ground permeability is known or can be estimated. A mathematical theory for estimating that runoff from topographic maps is derived.

Introduction

The purpose of this paper is do develop a simple mathematical model of ground flow so as to permit computing the long term steady state flow in rivers as the runoff component of the hydrological cycle. A fluid mechanical result is derived that is more exact than the Dupuit-Forchheimer theory of ground flow (1, 2), flow in a nearly flat field, but not as exact as the Muskat theory (3), which – by more exact integration of the Darcy equations of ground flow in terms of elliptic integrals – permits near infinite vertical "waterfall" flows, as required, for example, in seepage through dams. Those familiar with engineering results in elasticity will recognize that the solutions of the ground flow equations being proposed are equivalent to elementary elastic beam theory or plate theory in which stretching in the plane of the plate is allowed, as opposed to the bending of rods.

A hydrological model of a large land mass may be regarded as a two dimensional map of the land mass that has associated with it, at every point, exactly or in approximation, as many variables as are needed to characterize the total course of water on that land mass, and some laws or rules that connect these variables, point to point dynamically. The "independent" input to that model is the rainfall which has to be given by a model of the meteorological cycle.

Given slowly variable tectonic and plate erosion conditions, it can be assumed that large land mass surfaces present a sequence of dynamic topographic states that may be viewed as ergodic. That is, such an ensemble of states is equivalent to a large sequence of sand piles, each of which have been eroded by water from a distributed network of shower heads. The water streams create an ensemble of networks of rivers, one such pattern on each pile. While each network has an independent history, their statistics are comparable. It is to any one of these slowly changing geological states that a theory of runoff is to be applied.

Using the Earth's sea level geoid as a zero reference, each point on the large land mass can be indexed by planar, e.g., local Cartesian x, y or latitude-longitude sea level coordinates. To each such coordinate, a z coordinate, the height above sea level, can be indexed, thereby transforming the planar grid into an intrinsic ordinate topographic mapping of altitude coordinates associated with that planar x, y surface (a standard topographic map). Alternatively, a mapping orthogonal to those contours provide a map of "lines of steepest descent".

The nominal paths of the existing runoff system are along lines of steepest descent, according to the elementary law that water runs downhill in a gravitational field. But, because of the process by which rivers are formed, i.e., by land erosion and transport of a bed load, the paths of rivers meander among lines of steepest descent. The ongoing process of erosion and continental uplift continues to explore the ensemble of ergodic states that these river networks can occupy in long term.

Thus an intrinsic surface coordinate system for a continental land mass consists of the altitude contours and the not quite orthogonal paths of the river runoff

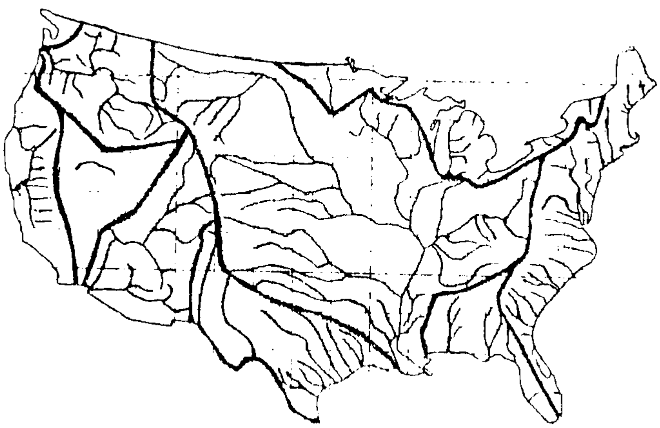

Fig 1 Major river basins and a river ordinate system for the United States (source [4, 5]).

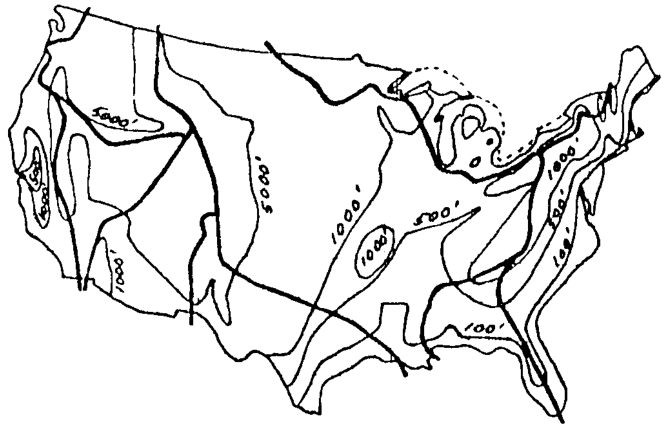

Fig 2 Major altitude contours in 1,000 feet intervals for the United States with a background of the river basins (source [4, 5]).

system. However, these systems are isolated in independent river basins, since the rivers do not cross ridges. Such a nominal coordinate system is illustrated for the United States in three figures – as isolated river basins and river ordinates (Fig 1), as altitude contours (Fig 2), and as the composite intrinsic coordinate system formed by the river net and altitude contours (Fig 3) – source (4, 5).

It is stressed that such an intrinsic map is not fixed, but – instead – it is the memory-laden resultant of yesteryear's events, both geological, e.g., tectonic, and meteorological, e.g., long term changing patterns of rainfall. Both these processes continue to modify the two coordinate systems. But, insofar as the variation is slow and typical of the class of all such mappings, then the map properties can be described by a near steady state characterization.

The steady state or nearly time-independent hydrological cycle field flow equation on such a land surface is

average precipitation = average runoff + average evaporation

This result is derived from the dynamic equation:

transient rate of change of ground water = short time averaged precipitation – runoff – evaporation

If this dynamic equation is integrated over a few years, assuming no geological changes and very little change in the water table, then the previous equation represents the time averaged steady state result. The equations stated here differ only from more standard sources in referring to an evaporation component rather than an evapotranspiration component. Our studies have indicated that, for long term-large area representations, the transpiration of plants represent negligible increment or decrement to the diffusive component of water surface evaporation (6, 7, 8). This point was argued out in great detail in (6). The essence of the argument is that the Bowen ratio for cloud covered regions rather quickly reaches a near constant value. That value represents the psychrometric constant of a wetted thermometer bulb, which the Earth thus resembles. Whether these results are accepted or not, the reader is free to translate the term evaporation to evapotranspiration without affecting the results in this paper which depends on other issues.

What is at issue in this paper is the question of causality. For example, referring to the mean state equation statement of the hydrological cycle, there are three terms. Independent causality can only be asserted for two of these terms. The question is thus which variable is not independent. This paper is based on the thesis that mean precipitation is an independent meteorological variable, and river runoff is an independent hydrological variable. Therefore, in this modeling, the evaporation is a mixed resultant of these two processes.

We have argued that the average river runoff is a characteristic associated with the existing river network, wherein that network is an indicator of the position of the water table at its breakout at river levels and the ground permeability to water flow. Thus the steady state, nearly time independent input to the drainage system is the average precipitation and the resultant "output" from the earth system is the evaporation.

As an illustrative approximation, without addressing causality, that balance can be approximatety exhibited as follows: Fig 4 illustrates the precipitation input to the United States as average inches of rainfall per year (source (9)). As estimated from that coarse grid, in which basically only 10 inch annual rainfall contours are shown, the mean rainfall in space and time is about 29.4 inches per year. The surface runoff at the boundary can be approximated by summing the flows from the principle river gaging stations for all the border states in contiguous continental United States. Using (5) as a source, the total runoff is estimated to be 1,835 thousand cu.ft./sec. Since there are 1.885 thousand million acres of

Fig 3 Intrinsic surface coordinate system net for the United States formed by river and altitude ordinates.

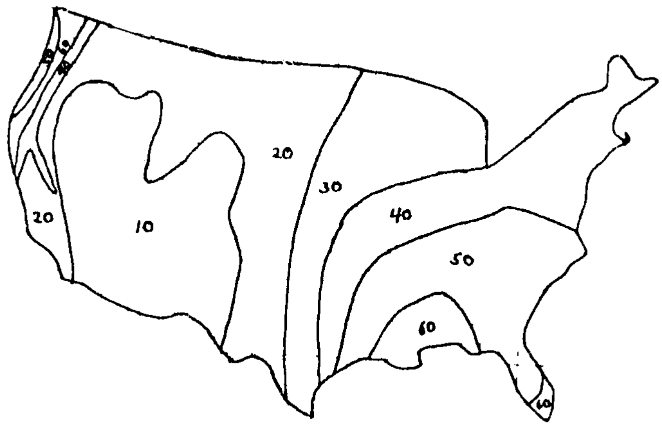

Fig 4 Mean annual precipitation in inches per year for the United States (source [9]).

land surface in contiguous United States, the net surface runoff is estimated to be about 8.5 inches of water. Thus the first approximate balance would provide an estimate of 29.4–8.5, equal to 20.9 inches of water evaporated per year.

A more detailed point by point estimate of evaporation (or evapotranspiration, as they call it) can be made from data prepared by the US Dept. of Agriculture (10). From their coarse-gridded mapping of evapotranspiration, an estimate of 19.1 inches of water per year was obtained. While an uncertainty of 2 inches of water can hardly be be regarded as precise, it suggests that the major details of the hydrological balance has been accounted for in an approximate overall fashion. Also we have shown, by a global theory for meteorological heat transfer, that the Earth's evaporation for cloud covered regions (e.g., those with more than 20 inches of rainfall per year) can be estimated to be about 23 inches of water per year (6, 7, 8), which is not incompatible with our more local balance for the United States.

I now propose to show a hydrological model of ground flow which permits one to estimate river runoff solely from such an intrinsic coordinate system of a large land mass like Fig 3. Its logic was outlined in (6, 7, 8) and is here summarized.

It is assumed that a river does not run along lines of steepest descent, but meanders among such lines. It is also assumed that large river systems are associated with river basins, which are defined by the ridge contours that separate adjacent basins. If a river meanders in its basin, then the basin can be divided into a series of narrow herringbone-like strips, which begin at nearby points on the centerline of the river within the basin and which follow lines of steepest ascent to the ridge contour. If it is assumed that surface water percolates into the ground in short space-time to join the water table, this as a time averaged process, which then also descends to follow the lines of steepest descent, then the flow drainage into the river from a slab whose surface strip is bounded by a segment of ridge contour, two lines of steepest descent, and an intersected segment of meandering river defines the increase in flow between the two intersected river stations. The drainage from that slab represents the average precipitation minus the evaporation for that strip of surface area. That is, the average increase in runoff in the marked segment of river is the net rainfall that percolated into the ground over that drainage strip associated with that river segment. The main theoretical idea is that the water table, representing the net run off flow between precipitation and evaporation, breaks out – by definition – at the river. Greater or lesser flows would break out above or below the river line.

A Mathematical Model of How Flow Percolating into the Ground Makes Up River Runoff

Geometry of the model. Start from a cross-sectional cut through a river valley, made in a vertical plane of any of the lines of steepest descent, extended at its flanks up to the ridges of the river valley. We will be concerned with a decomposition of the valley area into valley sections that are nominally of unit width, which will therefore be defined by a near parallel cross-sectional cut through another neighboring line of steepest descent. Two such vertical planes intersect two stations along the river in that valley. We wish to determine what net average rainfall (precipitation less evaporation) on that strip works its way into the river to increase the flow in that unit width section. By such a formulation, we have restricted ourselves to a two dimensional problem, involving x, the horizontal ordinate which measures the projected 'length' of the centerline of that small region which drains into the river segment (the projection of a line of steepest descent on the geoid plane), and z repre-

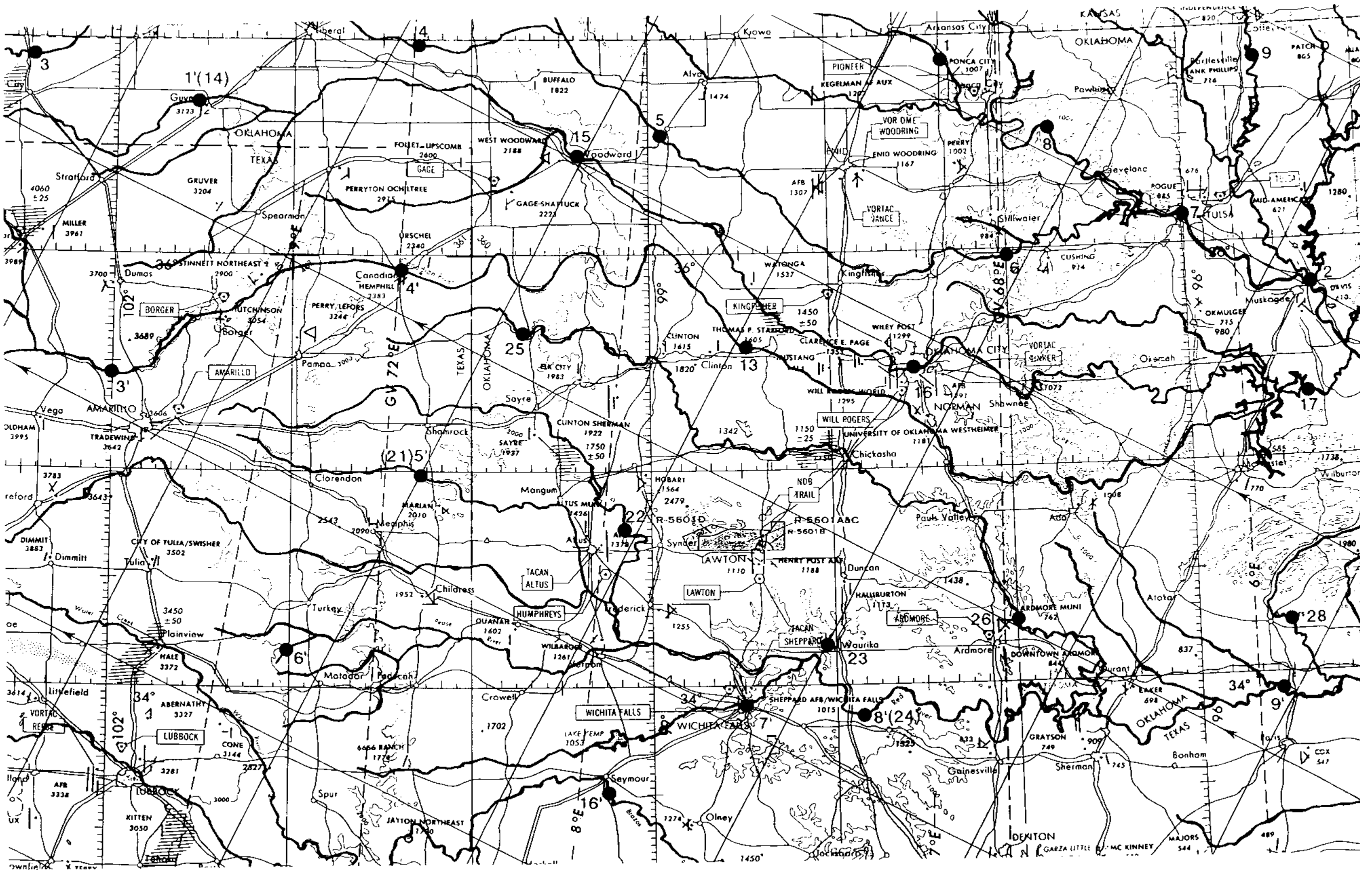

Fig 5 An illustrative river net (source [11, 12]).

sents the vertical ordinate along that small region of steepest descent. This x–z plane defines the plane of presentation for any characteristic unit ground flow problem. Actually, of course, the tortuousity of the terrain, which technically is the mathematical torsion measure, does not make the x–y projection of the line of steepest descent that cuts both the river and the ridge a straight line. That is, the coordinate system is not Cartesian. However, it is Cartesian in the limit, so that it requires the choice of the width unit to be small. It is by imagining the mean center of the the two adjacent lines of steepest descent laid out smoothly, so that x is really a measure of the more tortured line of steepest descent, that we have thereby isolated our two dimensional problem.

A hydological justification of the model. To permit the reader to assess the potential validity of these assumptions, five figures are used to illustrate a particular regional problem. The region considered had been chosen essentially at random.

In Fig 5, a river net is shown for an arbitrarily selected region in the United States, chosen at a particular scale of 1 to 2,000,000 (source (11)). The river stations identified on the map are taken from (12). The unprimed stations are in Oklahoma, the primed ones in Texas. In Fig 6, the river net and an altitude contour net are shown simultaneously (source of altitude data (11)). In Fig 7, the drainage basins for each of the major rivers is estimated. This was accomplished by smoothing altitude contours free of detail, constructing lines of steepest descent coordinates orthogonal to the altitude contours, paying no attention to the rivers. This thus separated the drainage basins between the rivers. To check the validity of our solution contours, we used a finer scale map of drainage basins (source (13)). The disparity in the estimated basin regions was small (see (6)). In Fig 8, the drainage areas associated with the various stations are shown. In Fig 9, the further decomposition of these drainage areas into 'infinitesimal' unit segments is shown. That decomposition shows that the drainage basins for river networks do not generally exhibit inordinately extreme tortuousities. Thus their developable Cartesian projection appears quite reasonable.

Mathematical formulation. In the mathematical formulation of the ground flow for such two dimensional slabs, we shall only solve for the drainage from one side of the river valley; in x–z coordinates, the river is represented as lying within a notched ditch or channel at the bottom of the river valley, of water height h_0. We assume that the average free surface of the ground water table, its return flow, breaks out at the river.

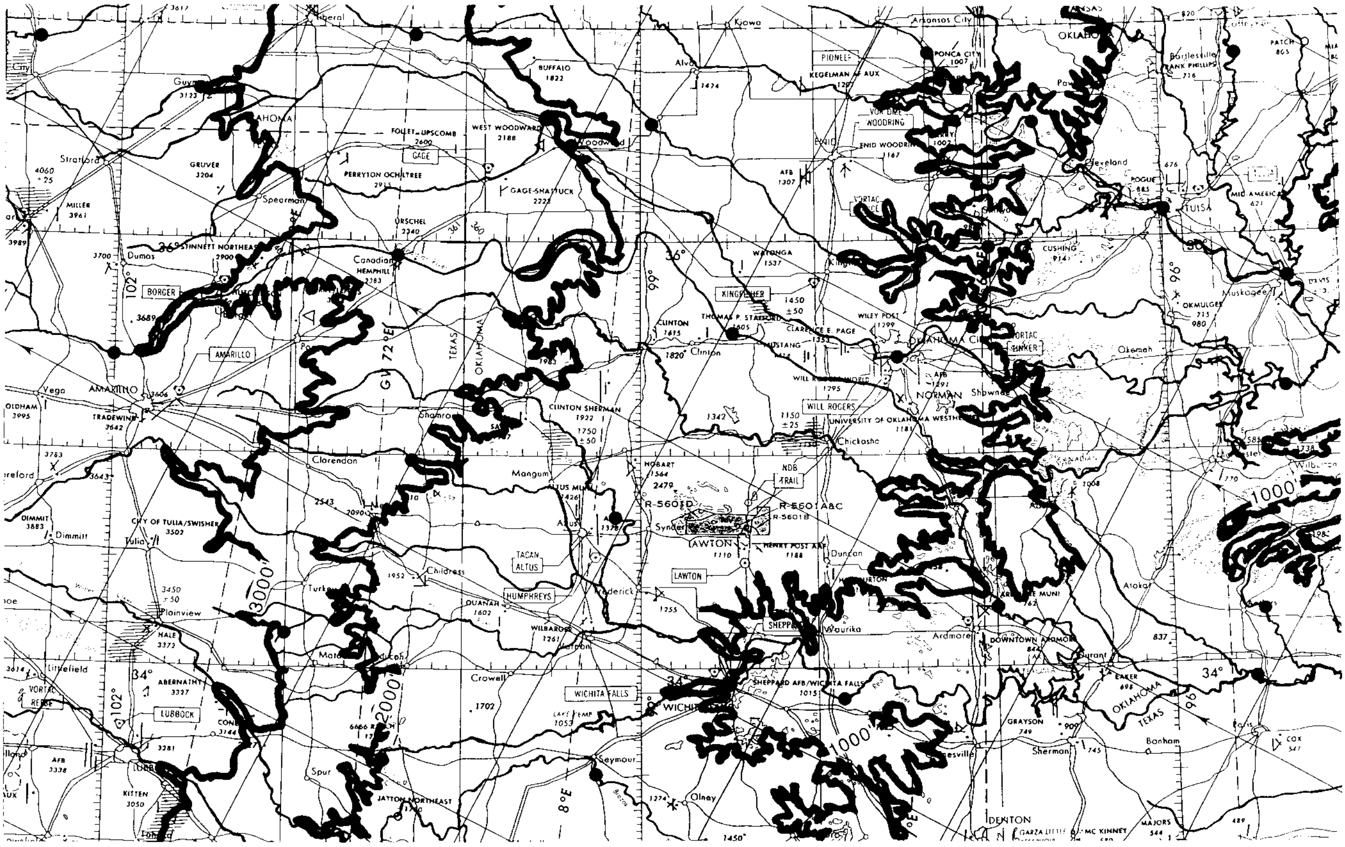

Fig 6 An intrinsic coordinate system – the river and altitude contour net (source [11]).

The ground over the entire strip of river valley, perpendicular to the plane of presentation, is charged by a net average intensity of rainfall w_o. For the sake of model simplicity, the net intensity is assumed constant over the entire strip of the river valley. That net rainfall, seeping into the ground, joins the water table at the free surface with a nominal boundary condition of hydrostatic pressure $p = 0$. That water table then drains, via ground flow, towards the river.

Another boundary condition for that drainage is that only the rainfall associated with that river valley segment drains to the river. Thus we can express the boundary condition by regarding there to be a horizontally impervious barrier perpendicular to the crest of the river valley. With these boundary conditions, we can set up and solve for the seepage flow field in the porous ground.

We will measure height z above a plane tangent to the bottom of the river section. We will measure distance x to the right from the rightmost bank of the river. The horizontal distance to the ridge crest will be measured by $x = L$. Hydrostatic pressure in the seepage field will be measured by p which is a steady state function of x and z.

The equations of motion in the seepage field are (6):

$$u = -k\,\partial p/\partial x \qquad [1]$$

$$w = -k\,(\rho g + \partial p/\partial z) \qquad [2]$$

$$\partial u/\partial x + \partial w/\partial z = 0 \qquad [3]$$

where u = horizontal velocity
w = vertical velocity
k = Darcy coefficient of permeability (conductance)

The equation of continuity [3], which represents two dimensional incompressible flow, implies that the pressure satisfies a Laplacian field

$$\partial^2 p/\partial x^2 + \partial^2 p/\partial z^2 = 0 \qquad [4]$$

The boundary conditions that we have to satisfy are the following:

1. $x = 0$, $p = \rho g\,(h_o - z)$
   On the left face of the slab, the pressure in the river increases hydrostatically with head.
2. $z = 0$, $w = 0$
   For practical purposes, at the plane level of the bottom of the river the vertical velocity is negligible in the river valley.
3. $x = L$, $u = 0$
   The river valley acts as if it is sealed off from an adjacent valley by a vertical barrier below the crest.

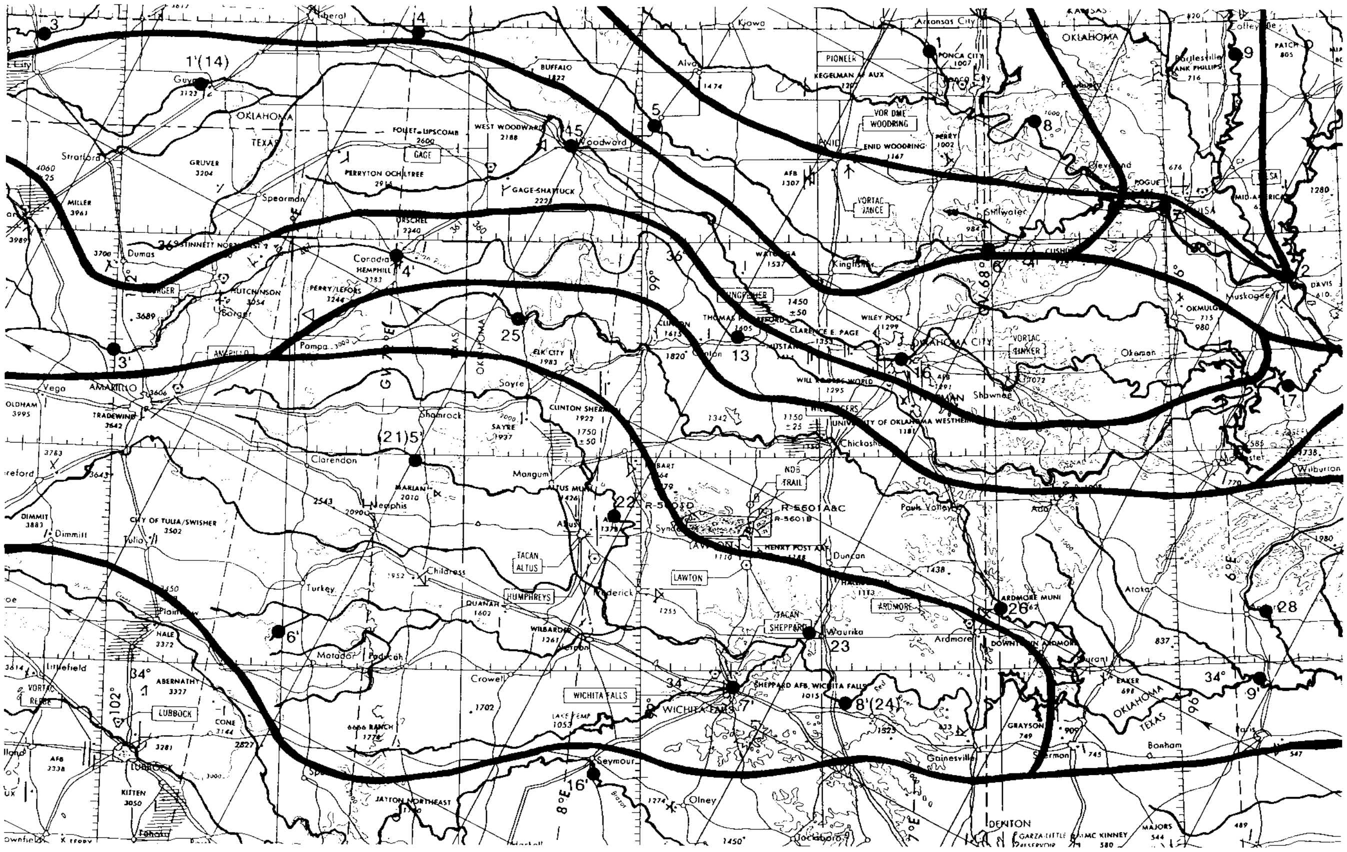

Fig 7 Estimated river basin drainage areas (estimated from Fig 5, 6; checked from source [13]).

4. $p = 0$, $z = h$: at $x = 0$, $h = h_o$; at $x = L$, $h = h_1$
 The free surface, defined by $p = 0$, is measured by its height z above the plane base.
5. $z = h$, $w = -w_o$
 The ground seepage is made up of the net vertical rainfall w_o which trickles into the free surface in the ground.

The basic assumption behind these boundary conditions is that the ground permeability is homogeneous and isotropic, and that there are no submerged aquifers bounded by water impervious strata which discharge into the river.

The most convenient form for p that we have been able to find is made up from

$$p = \rho g\ [h_o + F(x) + G(z) + z\ H_1(x) + z^2 H_2(x) + \cdots] \qquad [5]$$

where the various functions F, G, H_i have to satisfy Laplacian contraints. The form fits the large x, small z seepage field. It is simple to show that the first boundary condition is satisfied by

$$p=\rho g \left\{h_o+F(x)-F(0)+z[H_1(x)-H_1(0)-1]-z^2F''/2!-z^3H_1''/3!-z^4F^{(4)}/4!\cdots\right\} \qquad [6]$$

where

$$(F'')_{x=o}=(H_1'')_{x=o}=(F'''')_{x=o}=(H_1'''')_{x=o}=\cdots=0 \qquad [7]$$

The second boundary condition is satisfied by

$$p=\rho g\ [h_o+F(x)-F(0)-z-z^2F''/2!+z^4F''''/4!-z^6F^{(6)}/6!+\cdots] \qquad [8]$$

The third boundary condition is satisfied by

$$[F'-z^2\ F'''/2!+z^4\ F^{(5)}/4!-z^6\ F^{(7)}/6!+\cdots]_{\ x\,=\,L}=0 \qquad [9]$$

As a minimal necessary condition (e.g., a first approximation),

$$(dF/dx)_{\ x\,=\,L}=0 \qquad [10]$$

can be assumed.

The fourth boundary condition is satisfied by

$$h+h^2F''/2!-h^4F^{(4)}/4!+\cdots=h_o+F(x)-F(0) \qquad [11]$$

Various approximations, starting from

$$h=h_o+F(x)-F(0) \qquad [12]$$

can be assumed.

The fifth boundary condition is satisfied by

$$w_o/k\rho g=-hF''/1!+h^3/F^{(4)}/3!+\cdots \qquad [13]$$

Various approximations, starting from

$$hd^2F/dx^2=-w_o/k\rho g \qquad [14]$$

can be assumed.

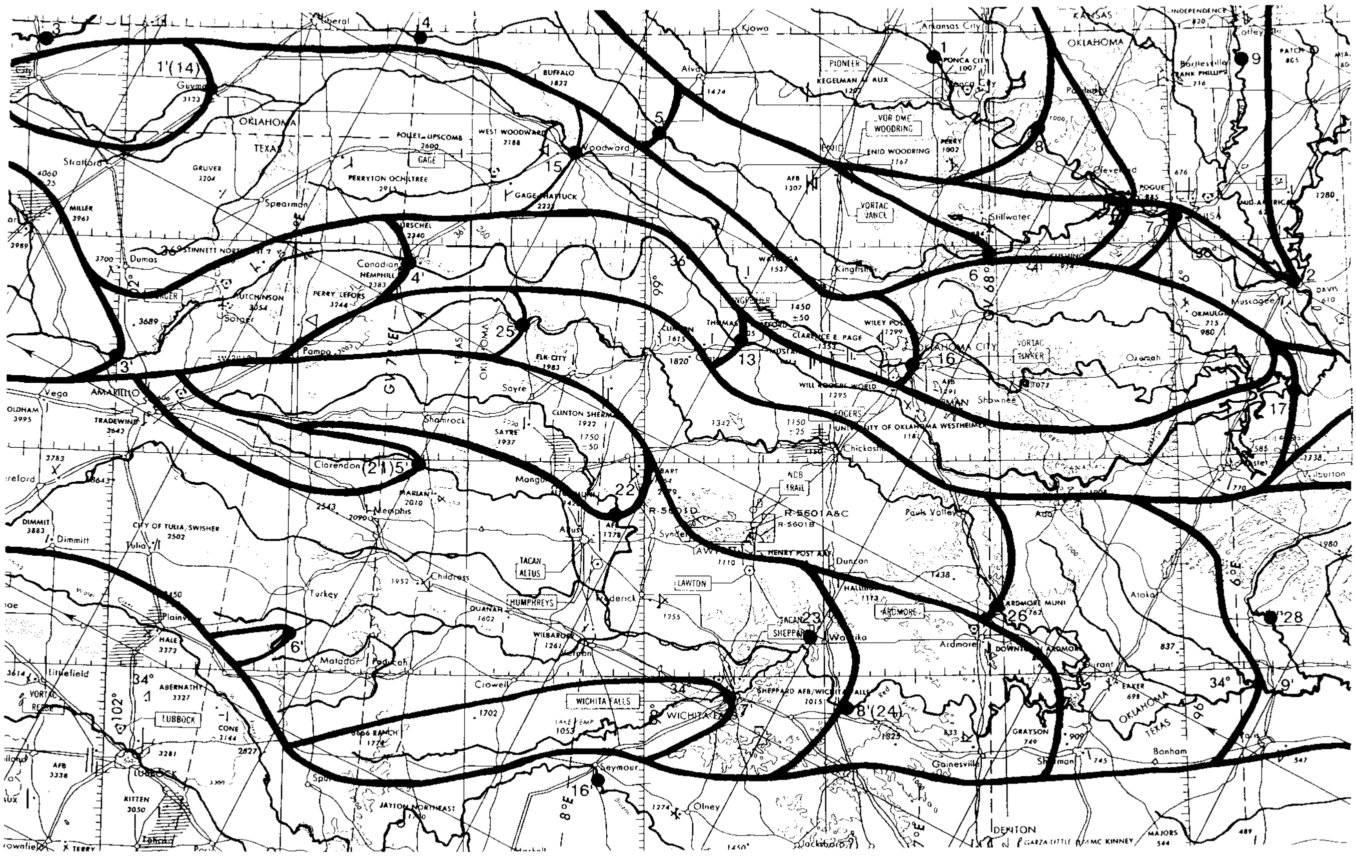

Fig 8 Drainage areas associated with river stations in Fig 5.

It seems clear that for a first approximation set, eqs. [10, 12, 14] are adequate. The parameter $w_o/k\rho g$ is small. Adding a quadratic term to h (namely h^2F'') only makes a negligible correction of magnitude $w_o/k\rho g$ to the form for h. The curvature in F'' (=h'') is basically small. Thus $h^2 F^{(4)}$ can be neglected compared to F''.

A first order, nearly flat approximation ($h_1 \approx h_o$) for this equation set gives

$$F = (w_o/k\rho g)\,(L/h_1)\,[x/L - x^2/2L^2]\,L \qquad [15]$$

$$h = h_o + (w_o/k\rho g\,(L/h_1)\,[x/L - x^2/2L^2]\,L \qquad [16]$$

$$(h_1 - h_o)\,L = (w_o/2k\rho g)\,(L/h_1) \qquad [17]$$

$$h_o' = (w_o/k\rho g)/(L/h_1) \qquad [18]$$

from which we obtain the result

$$w_o/k\rho g = (h_1/L)\,h_o' = 2\,(h_1/L)\,(h_1 - h_o)/L \qquad [19]$$

as a theory for the net average rainfall (i.e., the river runoff) as measured by w_o. The increment of increase of river discharge ΔQ, between two stations that are a distance Δy apart along the axis of the river valley, where $L\,\Delta y$ is the projected area of the catchment, is

$$\Delta Q/\Delta y = Lw_o = k\rho g\,h_1\,h_o' = h_1\,(k\rho g/2)\,(h_1 - h_o)/L \qquad [20]$$

This result, Eq. 20, represents a fairly exact statement of the computation that defines runoff in rivers. The relevant permeability-thickness product is kh_1 (where h_1 is the height of the water table above the river bottom at the crest of of the river valley. The estimate of incremental runoff can be made either from the initial slope h_o', or from the total river valley elevation $h_1 - h_o$, as $(h_1 - h_o)/2L$). Because the assumption can be made that the water table and the general ground slope are conformal, an assumption which is much better near the river than at the crests of the river valley, it would appear more precise to estimate the runoff from the initial slope than from $(h_1 - h_o)/2L$. However the difference is often only academic. Two notes on this point may be of some interest.

If the contour lines of steepest descent are drawn on the map of an actual mature river valley, following the river downstream, one finds the projection to be a herringbone pattern of near parabolic form (e.g., see Fig 9). The 'nose' of that steepest descent line corresponds to the active portion of the river valley in which the river frequently floods and meanders. If that portion is squared off, then we note that the catchment areas basically consist of parallelograms that drain into the flood area at a near constant angle (the asymptoptic flanks of the parallelograms). The imaginary x−z plane

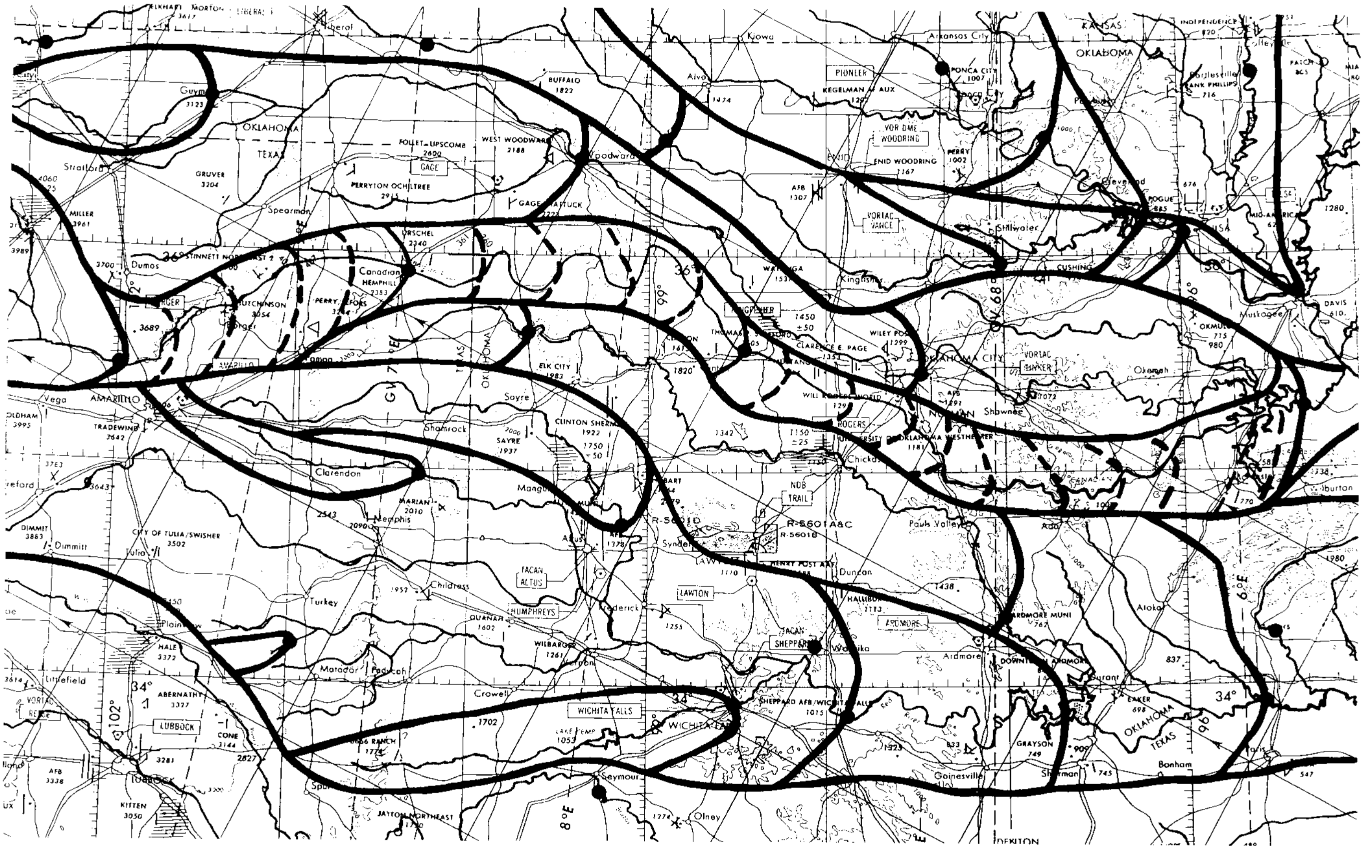

Fig 9 Illustrative detailed decomposition of drainage areas into small segments.

of presentation used to portray the flow field is thus drawn parallel to the sides of that parallelogram rather than drawn perpendicular to the axis of the river valley. There really is very little difference in how the flanks of the parabola are used to estimate the slope of differential height of the drainage basin. The uncertainty can be reduced to appreciably less than the factor of two that appears in the two different forms of computation of slope.

As a second note, the nature of the erosion that created the system of river valleys suggests that the form of a mature river valley of extended size approximates the shape that the water table would have in this theory of ground flow. Namely, as we stated, the ground flow is conformal with the water table. This would be born out by the observation that even though we represented a river valley with a single 'central' channel, this valley included extensive stream systems in which local steam valleys exist at many scales on the flanks. We can summarize this property by two statements: One that yesteryears rains, as average runoffs, wear the lands down conformally to its current shape including the current rivers they mark on the flanks of existing valleys.

The model prototype for that conformality is the process of shaping hard surfaces by lapping abrasion, e.g., by the eroded river bed load, such as that used in forming optical mirrors. The speed of such abrasion, in geological time, is routinely observed almost daily by visitors in the wear of the Niagara Falls by the Niagara River. Even when plate movements are sufficient to create waterfalls, for example in rift valleys, the wearing process works to make the surface somewhat conformal to the water table. That is, mature landscapes exhibit broad gentle river valleys created by the actions of the surface, the underground, and the bed load erosion.

For our model result, we need a little more security than the flat approximation ($h_1/h_o \approx 1$). Returning to the equation set and the next approximation, the constraint equations can be written, in terms only of h, as

$$h\, d^2h/dx^2 = -w_o/k\rho g \qquad [21]$$

$$(dh/dx)_{x=L} = 0 \qquad [22]$$

from Eqs. [9–14]. The solution to this pair of equations is

$$dh/dx = h_o' \left\{ (\ln h_1/h)/(\ln h_1/h_o) \right\}^{1/2} \qquad [23]$$

$$h_o' = \left\{ 2w_0/k\rho g] [\ln h_1/h_o \right\}^{1/2} \qquad [24]$$

The substitution

$$\ln h = \ln h_1 - t^2/2 \qquad [25]$$

reduces the solution of the differential equation to the area under a Gaussian normal error curve. Thus

$$N(t_a)-N(t)=(1/2\sqrt{\pi})[h_o'/(\ln h_1/h_o)^{1/2}]\ [L/h_1]\ [x/L] \quad [26]$$

$$t=[2\ln (h_1/h)]^{1/2},\ t_o=[2\ln (h_1/h_o]^{1/2} \quad [27]$$

where N(t) is the area under the Gaussian normal error curve $\{N(0)=0,\ N(\infty)=0.5\}$, given by

$$N(t)=\frac{1}{\sqrt{2\pi}}\int_o^t \exp(-t^2/2)\,dt \quad [28]$$

Whereas in the flat approximation

$$w_o/k\rho g = 2(h_1/L)\,[(h_1-h_o)/L] \quad [29]$$

in the full approximation

$$w_o/k\rho g = 2m(h_1/L)\,[(h_1-h_o)/L] \quad [30]$$

h_1/h_o	m
1	1
1.1	0.987
1.5	0.939
2.718	0.882
∞	0.785

where

$$m=(h_1/h_o)\pi/(h_1-h_o)\left\{N(t_o=[2\ 1n\ h_1/h_o]^{1/2})\right\}^2 \quad [31]$$

Considering the crudeness of the data available to model the Earth, it is sufficient to use some approximate measure such as

$$w_o/k\rho g=1.6\ (h_1/L)\ (h_1-h_o)/L \quad [32]$$

Therefore

$$\Delta Q/\Delta y=0.6\ k\rho g\ [(h_1-h_o)/L]\ h_1 \quad [33]$$

introduces a minor refinement over the earlier estimate.

The ground seepage into the river per unit drainage area $\Delta Q/L\Delta y=q$, is thus related to the Darcy conductance k by

$$q=0.6\ k\rho g\ h_1(h_1-h_o)/L^2 \quad [34]$$

or

$$0.6\ k\rho g=\left\{L^2/[h_1(h_1-h_o)]\right\}q \quad [35]$$

We have applied this relationship to a large river segment, for example to all the available data from flow stations along 2/3rds of the Missouri River (6, 7). From data on the various drainage basins and the increments of river flow from river station to river station, we have computed the average ground permeability to be about 1.2×10^{-6} cm^2, in which water viscocity at ntp is assumed. We estimated the standard deviation of our unit area permeability estimates to be about 40%. Such a ground permeability is on the order of what might be expected from loose beds of sand, which may be considered to be a plausible estimate.

Rather than only using estimates of ground permeability, we believe that satellite pictures, taken for example on Earth, could make observations of the unit hodograph transient runoffs after rainfalls, and likely be used to make fair estimates of local ground permeabilities.

References

1) Freeze, R.; Cherry, J.: Groundwater. Prentice-Hall Inc., Englewood Cliffs, NJ 1979.
2) Fetter, C.: Applied hydrology. Charles E. Merrill Publishing Co., Columbus, OH 1980.
3) Muskat, M.: The flow of homogeneous fluids through porous media. MsGraw-Hill, NY 1937.
4) United States contour map, US Dept. Interior, Geological Survey, edition of 1916, reprinted 1957.
5) Langbein, W. et al.: Annual runoff in the United States. US Geological Survey, Circular 52, 1949, reprinted 1959.
6) Iberall, A.; Cardon, S.; Schneider, H.: A study for the physical description of the hydrology of a large land mass pertinent to water supply and pollution control. Four quarterly reports for DWSPC, Public Health Service, Dept. Health, Education, and Welfare, Washington DC, Contract SAph 78640 (division currently in the Dept. of the Interior), 1961–1962.
7) Iberall, A.; Cardon, S.: Contributions to a thermodynamic model of Earth systems. NASA Contractors Reports, Contract NASW-3378, prepared by Gen. Tech. Serv. Inc., May, Aug., Nov. 1980, Jan. 1981 for NASA-Washington.
8) Iberall, A.: On rivers. In: Yates, F. (ed.), Self-organizing systems, pp. 33–47, Plenum Press, NY 1987.
9) Mean annual precipitation. National Atlas of the United States, 1970, US Dept. Interior, Geological Survey, Washington DC.
10) Extended water requirements for agricultural purposes and their effects on water supplies. Committee print No. 13 of Select Committee on National Water Resources, US Senate, Jan. 1960.
11) Jet navigation maps. US Dept. Commerce, Coast and Geodetic Survey JN 44. Also see JN 29, 30, 45.
12) US Geological Survey Surface Water Resources of the US Committee print No. 4 of Select Committee on National Water Resources, US Senate, Jan. 1960.
13) US Geological Survey Map of US showing drainage areas and Index Nos. Plate 1. Water Supply Paper 1360, 1956.